CHAPTER 8

Parametric Design

LEARNING OBJECTIVES

When you have completed this chapter you will be able to

- Describe design phase information inputs/outputs
- Specify solution evaluation parameters
- Identify and characterize design variables
- Establish explicit and implicit constraints
- Select and use methods to analyze alternative designs
- Develop parametric spreadsheets to refine designs
- Characterize and assess overall customer satisfaction
- Explain design for robustness

8.1 INTRODUCTION

Parametric design includes a number of decision-making processes, just like the other phases of design. The processes use information (input) from prior phases to arrive at logical decisions (output). What makes parametric design special and particularly challenging is that we will employ analytical and experimental methods to predict and evaluate the behavior of each of the design candidates to make these decisions.

Let’s examine the types of input and output information that we will process as we make our decisions in the various design phases.

8.1.1 Information Flow in the Design Phases

As we recall the phases of design as shown in Figure 8.1, information about the customers’ needs is processed in the formulation phase usually resulting in a list of customer requirements/product functions, the importance of each, a list of engineering characteristics that quantitatively describe how well the functions are performed, a detailed engineering design specification, and ideally, a House of Quality diagram.
We use that information as input to the concept design phase, wherein we make decisions about the physical principles, abstract embodiment, primary manufacturing processes, and material classes.
Similarly, we use that information as input to the configuration design phase, wherein we make decisions about the product’s architecture to decide which parts should be standard or special purpose. Then we determine special-purpose parts geometric features, their arrangement and/or connectivity, and their relative dimensions. And we establish a list of attributes or variables that we will need values for. And for standard parts, we select the specific type and attributes that we will need values for.

In the **parametric design** phase, we determine the values of those attributes, typically called **design variables**. These usually relate to specific sizes, lengths, radii, diameters, material types, and manufacturing process requirements. Each set of design variable values is called a design candidate. We predict the performance of alternative candidates using analytical methods and/or experimental methods. If the performance of the candidates satisfies all the design constraints, we call them **feasible designs**. Then we evaluate the feasible designs to determine which candidate is the best.

Finally, during the detail design phase, we complete the remaining decisions, resulting in comprehensive product specifications, drawings, manufacturing specifications, performance tests, and bills of materials. Next, we examine the parametric design of a simple flange bolt.

### 8.1.2 Parametric Design: Pipe Flange Bolt Example

At a petrochemical plant, eight bolts will be used to fasten a cooling water pipe to the inlet flange of a high-pressure chemical reactor vessel. The piping system is subjected to an operating pressure of 1,000 psi. The flanges will be subjected to temperatures up to 250°F. Each bolt will be required to withstand a 4,000 pound design overload, which is four times the typical operating load of 1,000 pounds. The company has suggested a standard hex-head bolt, shown in Figure 8.2. Some of the design variables for bolts include: bolt diameter \( d \), overall length \( L \), number of threads per inch, and type of material. Assume that for this example, we have been asked to determine the bolt diameter only.

The principal function of a bolt is to fasten parts together so that they can be disassembled later for maintenance or repair. As the bolt is tightened, a tensile force develops over its grip length. The greater the force, the better the clamping action. Also, since the head of the bolt consequently develops higher friction forces, there is an improved resistance to loosening.
However, a bolt can fail to function in a number of ways, including: (1) the bolt head can twist off during tightening, (2) the bolt threads can strip off, and (3) the bolt can experience excessive tightening or excessive operating loads that it permanently elongates or pulls apart.

Let’s consider only the last mode of failure for this example. The ability of a bolt to withstand the tensile force without a permanent measurable elongation or “set” is called the proof load \( F_p \), and is directly proportional to cross-section area \( A \) of the bolt and the material’s proof strength \( S_p \), by the relation:

\[
F_p = A S_p \tag{8.1}
\]

The Society of Automotive Engineers (SAE) has categorized proof strengths of bolt materials as Grades 1–9, with strengths ranging from 33,000 psi to 120,000 psi, respectively. Assume that we select a Grade 5 material, whose proof strength is 85,000 psi.

Therefore, for this design problem, we need to design the bolt strong enough to have a proof load that is greater than the 4,000 (lbf.) design load. This is a constraint for this problem. Consequently, we need to find an area and, therefore, diameter such that:

\[
F_p \geq 4000 \text{ lbs} \tag{8.2}
\]

By substituting equation (8.1) into the constraint equation (8.2), we obtain:

\[
A S_p \geq 4000 \text{ lbs} \tag{8.3}
\]

We find that that the proof-load constraint can be satisfied if the area:

\[
A \geq \frac{4000}{S_p} \tag{8.4}
\]

\[
A \geq \frac{4000 \text{ (lbs)}}{85,000 \text{ (lbs/in}^2\text{)}} \tag{8.5}
\]

\[
A \geq 0.047 \text{ in}^2 \tag{8.6}
\]

and since cross-sectional area is related to diameter \( d \) as:

\[
A = \frac{\pi d^2}{4} \tag{8.7}
\]

substituting, we find that

\[
\frac{\pi d^2}{4} \geq 0.047 \text{ in}^2 \tag{8.8}
\]

and further, that

\[
d^2 \geq \frac{0.047 (4)}{\pi} = 0.0598 \text{ in}^2 \tag{8.9}
\]

\[
d \geq 0.245 \text{ in.} \tag{8.10}
\]
The calculated diameter is then rounded up to the nominal size 0.25 in., so that a standard size could be purchased.

As shown in Figure 8.3, as we increase the diameter, the calculated proof load increases exponentially. The required proof load is constrained to be equal to or greater than 4,000 (lbs.) and is shown as a horizontal line. The minimum feasible diameter is shown as the dashed line at 0.245 (in.) and, therefore, satisfies the constraint. Selecting a diameter larger than 0.245 (in.) would improve the safety and reliability of the design and could be considered in the final design.

This parametric design problem is rather simple, in that we have only one governing constraint (designed proof load must be greater than 4,000 [lbs.]) and one algebraic relation to substitute for the area of a circular cross section. Therefore, we can directly solve for the diameter that satisfies the constraint. There is no need to iterate. Some engineers call this type of problem inverse analysis, because we can rearrange an analysis equation(s) to find the unknown value. Analytical relations are often derived from experiment as function = \( f(\text{form}) \). By inverting the analysis equation we find \( \text{form} = f^{-1}(\text{function}) \).

The parametric design of most mechanical parts is much more difficult. Let’s examine the overall process to figure out how we can be more systematic, and thereby complete better designs with less effort.

### 8.2 STEPS IN SYSTEMATIC PARAMETRIC DESIGN

Systematic parametric design has five major steps, as shown in Figure 8.4:
Step 1: Formulate the parametric design problem.
Step 2: Generate alternative designs.
Step 3: Analyze/predict the performance of the alternatives.
Step 4: Evaluate the performance of each alternative.
Step 5: Optimize/refine.

Step 1. Formulate the problem. Practicing engineers often comment that they spend 40–50 percent of their time gathering appropriate data, clarifying important details, considering different analytical and experimental analyses, and then planning how they will complete a project. All these activities relate to problem formulation. As we start a parametric design problem, we need to familiarize ourselves with the problem parameters and also plan ways complete the design.
Solution Evaluation Parameters (SEPs). By reviewing the House of Quality chart and the engineering design specifications we reaffirm what functions the customer wants the product to perform. We then select engineering characteristics to measure the predicted performance of the functions. Solution evaluation parameters are the engineering characteristics that are selected to determine how well a candidate design “solves” the problem (Dixon and Poli, 1995).

Solution evaluation parameters depend upon the product and part being designed, but often include: cost, weight, speed, efficiency, safety, and reliability. We also denote parameter symbols, units (of measurement), and any lower and or upper limits of the parameter. As mentioned in Chapter 3, we can also describe the customer’s satisfaction with respect to each solution evaluation parameter, as a satisfaction curve. Finally, we agree on analytical or experimental methods to determine their values.

In the flange bolt example, the customer wants the bolt to clamp the pipe flange to the pressure vessel flange. The clamping force can be measured by the proof load and is, therefore, an example of a solution evaluation parameter. The higher it is, the better the clamping force, and the more satisfied the customer. We selected \( F_p \) as its symbol and pounds-force as the units. A lower limit, 4,000 (lbf.), is also established. The bolt proof load is constrained to be larger than 4,000 (lbf.). An analytical formula will be used to predict the proof load, although experimental tension tests could have been used.

Design Variables (DVs). Parameters under the control of the designer, which influence the candidate’s performance, are called design variables. Design variables usually relate to part dimensions, tolerances, and/or material properties. In addition to the design variable names, we establish appropriate symbols, units, and upper and/or lower limits or bounds. For discrete variables we determine permissible values (e.g., 2 by 4, 2 by 6 lumber, \( \frac{1}{2} \) inch diameter bolts).

The diameter is selected as the design variable in the flange bolt example. Larger diameters result in larger areas and consequently, improved performance. Permissible values are identified using handbook data.

Problem Definition Parameters (PDPs). Parameters that describe specific conditions of use, such as operating conditions are called problem definition parameters. In addition to their names, we establish appropriate symbols, units, and values.

The operating pressure of 1,000 psi and the limiting temperature of 250° F are identified as PDPs for the flange bolt example.

Preliminary Plan for Solving the Problem. For small design problems, we often jump right in and start calculating things. For larger, more involved problems, we need to make a preliminary plan based on considerations including:

1. Do we have analytical models/formulas for our problem?
2. Are the assumptions used in our models the same as our problem?
3. Will we need to perform pilot scale or bench-top experiments to validate our analytical formulas?
4. How much time and money do we have to “solve” the problem?
5. Are “ballpark” computations required, or do we need more thorough and precise calculations?
6. Do we have knowledge about acceptable industry standards?
7. Will manufacturing specifications need to be generated from our analyses?
8. Do we understand the customers’ function requirements versus satisfaction well enough?
9. Are we sufficiently qualified or competent to complete the required design?

For large design problems, design teams will consider the questions above and will prepare a design project proposal for upper management to review and approve. Design project proposals include items such as a background section, goals or mission of the project, a scope of work (of the work tasks to be performed), schedule, and budget. Regardless of the size of your project, however, it is always a good idea to prepare an outline of what you are going to do and when and how you are going to do it.

Recognizing the simple nature of the flange bolt calculations, we went straight to making a few calculations.

**Step 2. Generate alternative designs.** We select different values for the design variables to generate different candidate designs. These values can come from our own experience, from our company’s experience, or from industry standards. Sometimes we need to make educated guesses.

**Step 3. Analyze the alternative designs.** We predict the performance of each candidate design using analytical and/or experimental methods:

**Analytical Methods.** Formulas from physics, mathematics, and the engineering sciences are most often used. Sometimes, advanced computer-aided-design packages, such as finite element analysis, computational fluid dynamics, and motion simulation, are used.

**Experimental Methods.** Oftentimes, the complexity of the design is beyond the accuracy or assumptions of our analytical models. In these cases, we can build scale models and/or full-scale models of the product or critical parts of the product, and test their performance. We can use wind tunnels, for example, to analyze the performance of complicated wing geometries, or check control surfaces.

The performance of each design candidate is checked so that every performance constraint is satisfied. These designs are called feasible designs. If the constraints are violated, we reiterate back to generating another alternative and then analyzing it.

As we substitute new values for the design variables into the system of analysis equations we become familiar with how each design variable influences part or product performance. Using this familiarity, we can sometimes generate new values that satisfy the constraints. Dixon and Poli (1995) call this
“physical reasoning.” In other words, by becoming familiar with the causes and effects, we can logically reason the “physics” of the problem.

If no feasible design candidates exist, we select new values for the design variables, and thereby generate new design candidates. These are subsequently analyzed for feasibility. This is the solid “redesign” iteration loop shown in Figure 8.4. If we cannot find any feasible candidates, we might ask whether we have set our design specifications too restrictively. Perhaps one or more constraints could be relaxed. If so, we are “respecifying” as shown in Figure 8.4.

The formula for the area of a cylindrical cross section (8.7) and the formula relating the proof load of a bolt to its area and strength (8.1) were used in the bolt example. The performance constraint considered in the bolt example was that the proof load be larger than 4,000 (lbs.). Since the formulas were straightforward, we were able to juggle the equations into a sequence that did not require iteration. More complicated problems will not be “solvable” by equation juggling.

**Step 4. Evaluate the results of the analyses.** The feasible designs are evaluated to determine the best design. Usually one or more criteria are identified in the formulation phase and used to determine the “best” feasible design alternative.

The proof load was established during formulation as a rough measure of customer satisfaction. We assume that the higher the proof load, the more satisfied our customer would be. That would mean that a large bolt should be chosen as the “best” design. This, however, ignores other aspects of real design problems such as weight and cost limitations. Unfortunately, since we do not know how the customer feels about these issues, we can only surmise that he might be satisfied with the smallest bolt that meets the force constraint. As we shall see in the next section, we should always try to ascertain customer satisfaction with respect to each solution evaluation parameter.

**Step 5. Optimize/refine.** Optimal design methods automatically regenerate new values of the design variables to improve expected performance and satisfaction (Arora, 1989; Papalambros and Wilde, 1989; Rao, 1984; Reklaitis et al., 1983; Siddall, 1982; Vanderplaats, 1984). For single-attribute optimization, a single criterion is chosen, such as minimizing the weight of a part. The criterion is a function of the design variables, and is called the **objective function**. Design variable values are usually constrained to some upper and lower limits. The predicted performance of a part is also related to the design variables. For example, the strength of a bolt is a function of its diameter. Design variable functions and limits are called **constraints**. Equation (8.2), for example, is an inequality constraint. Excel’s Solver feature can be used to optimize many typical design problems found in mechanical engineering. This is shown as the dashed regenerate loop in Figure 8.4.

A more detailed example using this systematic procedure is presented in the next section.
8.3 SYSTEMATIC PARAMETRIC DESIGN: BELT-AND-PULLEY EXAMPLE

Let’s apply the systematic procedure presented in the last section using a more detailed example of a belt-and-pulley transmission.

8.3.1 Design Problem Formulation

A ½-hp electric motor, running at 1,800 rpm, will be used to drive a grinding wheel operating at 600 rpm. A flat belt-and-pulley drive system configuration has been selected, as shown in Figure 8.5. The design team has also determined that:

- the drive motor will have a 2-in. diameter pulley mounted,
- candidate designs should be able to transmit the full horsepower,
- the customer desires a compact system design,
- the drive pulley will slip first, before the driven pulley,
- the purchasing department has located a vendor that can provide a flat belt that can withstand a maximum 30-pound tensile load,
- the coefficient of friction between the belt and pulley is 0.3,
- other design engineers in your group will design the mountings, bearings, and protective equipment, therefore,
- parametric design efforts should focus on the distance between centers and the diameter of the driven-pulley.

![Motor Pulley (driver) \( r_1, d_1, \phi_1, n_1 \) and Grindling Wheel Pulley (driven) \( r_2, d_2, \phi_2, n_2 \)](image)

FIGURE 8.5 Belt-and-pulley-drive system for motor and grinding wheel.
**Solution Evaluation Parameters (SEPs)** The principal function of the pulley is to transform the power of the motor from a high speed to low speed. In the transformation, the smaller motor torque is converted to a larger torque, according to the conservation of energy law. Also, the belt-and-pulley system would fail to perform its principal function if the belt slipped or if the belt broke owing to excessive tension. Finally, the customer would be more satisfied with a compact design.

Since we know that the tension force in the belt is limited by the amount of friction between the belt on the driver pulley, up to the point of impending slip, we could determine the torque that the belt can deliver to the pulley, \( T_b \), and compare it with the maximum torque, \( T_m \), that the motor can supply. Also, we should calculate the maximum belt tension, \( F_1 \), to make sure that it does not exceed the 35 (lbs.) limit.

We can summarize the solution evaluation parameters in Table 8.1.

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Symbol</th>
<th>Units</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 belt torque</td>
<td>( T_b )</td>
<td>lb-in</td>
<td>( T_m )</td>
<td>—</td>
</tr>
<tr>
<td>2 belt tension</td>
<td>( F_1 )</td>
<td>lbs</td>
<td>—</td>
<td>35</td>
</tr>
<tr>
<td>3 center distance</td>
<td>( c )</td>
<td>in.</td>
<td>small</td>
<td>—</td>
</tr>
</tbody>
</table>

**Design Variables (DVs)** The value of the center distance, \( c \), directly affects the compactness of the design and is to be determined by the designer. Also, as the center distance is increased, more of the belt wraps around the pulley (i.e., is in contact with the surface of the pulley), increasing the ability of the belt to grip the pulley and thereby satisfy the torque requirements of the motor. The design variables are summarized in Table 8.2.

<table>
<thead>
<tr>
<th>Design Variable Description</th>
<th>Symbol</th>
<th>Units</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 center distance</td>
<td>( c )</td>
<td>in.</td>
<td>small</td>
<td>—</td>
</tr>
<tr>
<td>2 driven pulley diameter</td>
<td>( d_1 )</td>
<td>in.</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

**Problem Definition Parameters** Studying the design problem data, we find a number of “givens” that define design problem conditions such as the friction coefficient, belt strength, motor power, and motor pulley diameter. Therefore, we identify these as the problem definition parameters in Table 8.3.
TABLE 8.3 Problem Definition Parameters for Belt-Pulley System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 friction coefficient</td>
<td>$f$</td>
<td>none</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>2 belt strength</td>
<td>$F_{\text{max}}$</td>
<td>lbs</td>
<td>—</td>
<td>30</td>
</tr>
<tr>
<td>3 motor power</td>
<td>$W$</td>
<td>hp</td>
<td>—</td>
<td>½</td>
</tr>
<tr>
<td>4 motor pulley diameter</td>
<td>$d_1$</td>
<td>in.</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

**Plan for Solving the Design Problem** Using analytical relations from physics and mathematics we can use a hand calculator or develop a spreadsheet to calculate the engineering characteristics, including:

1. grinding wheel pulley speed, $n_2$,
2. angle of wrap as a function of the center distance, $c$,
3. belt torque, $T_b$,
4. maximum belt tension, $F_1$,
5. slack-side belt tension, $F_2$, and
6. initial tension (before torque is applied), $F_i$.

Then we will check that the constraints are not violated. Specifically, we will make sure that the belt will deliver the full motor torque to the grinding-wheel pulley and that the belt tension does not exceed the belt strength limit.

**8.3.2 Generating and Analyzing**

Following our plan, we need to develop an analytical means to predict the behavior of the system. We can model the behavior of the system using relations from physics and mathematics and develop a system of equations to analyze the performance of each candidate design as we substitute different values for the design variables.

For example, we know that a motor will deliver power $W$ (hp), to a pulley rotating at $n$ rpm when producing a torque $T_m$ lb. ft. according to equation (8.11):

$$ W = \frac{T_m n}{5252} \quad \text{(hp)} \quad (8.11) $$

If the belt is not permitted to slip on the pulleys, the pulley speeds are related to the ratio of the pulley diameters.

$$ \frac{n_2}{n_1} = \frac{d_1}{d_2} = \frac{r_1}{r_2} \quad (8.12) $$

We can determine the angle of wrap $\phi$ using basic geometric relations.
\[ \phi_1 = \pi - 2 \sin^{-1}\left( \frac{d_2 - d_1}{2c} \right) \]  

(8.13)

Similarly, we know that a belt, with coefficient of friction \( f \) that is in contact with the pulley for an angle of wrap \( \phi_1 \), has a maximum belt tension \( F_1 \), on the taut side of the pulley, and is related to the belt tension \( F_2 \) on the slack side of the pulley according to:

\[ \frac{F_1}{F_2} \leq e^{f \phi} \]  

(8.14)

Upon sketching the free-body diagram, shown in Figure 8.6, we see that for static equilibrium, we can sum the moments about the bearing \( B \) to obtain the torque \( T_m \) delivered by the belt to the driver pulley of radius \( r_1 \), as:

\[ T_b = (F_1 - F_2)r_1 \]  

(8.15)

As a point of interest, we can show how the angle of wrap affects the torque capacity of the belt at impending slip, by substituting equation (8.14) into (8.15). By examining the resulting equation (8.16), we see that as the angle of wrap increases and the torque increases.

\[ T_b = (F_1 - F_2)r_1 = \left( F_1 - \frac{F_1}{e^{f \phi}} \right)r_1 = F_1 \left( 1 - \frac{1}{e^{f \phi}} \right)r_1 \]  

(8.16)

The maximum torque delivered by the motor, \( T_m \) can be determined by rearranging equation (8.10) as:

\[ T_m = \frac{W 5252}{n} = 0.5 \left( \frac{5252}{1800} \right) = 1.46 \text{ (ft lbf)} = 17.52 \text{ (in. lbf.)} \]  

(8.17)
The belt must be able to provide a torque equal to or greater than 17.52 (in. lbf.); otherwise it will slip.

Using equation (8.12) we can predict the driven-pulley speed, \( n_2 \), as a function of the design variable, diameter \( d_2 \).

\[
n_2 = n_1 \frac{d_1}{d_2} = 1800 \frac{2}{d_2} \text{ (rpm)} \tag{8.18}
\]

Note that equation (8.18) is an equality constraint, having one unknown design variable, \( d_2 \). A feasible candidate design must satisfy this constraint. Since the grinding-wheel speed has been specified as 600 rpm, we can obtain a feasible value of the driven-pulley diameter by rearranging equation (8.18).

\[
d_2 = d_1 \frac{n_1}{n_2} = 2 \left( \frac{1800}{600} \right) = 6 \text{ (in.)} \tag{8.19}
\]

Therefore, the only feasible value of the design variable \( d_2 \) is 2 in.

**Generating an Initial Value of the Design Variable**

Since the customer will be more satisfied with a compact design, we would like the distance between the pulley centers, \( c \) to be small. The closest that the two pulleys can be is when their radii are almost touching, theoretically speaking, or

\[
c_{\text{min}} = r_1 + r_2 = \frac{d_1 + d_2}{2} = \frac{6 + 2}{2} = 4 \text{ in.} \tag{8.20}
\]

Now we can use equation (8.13) to find the angle of wrap on the driving pulley.

\[
\phi_1 = \pi - 2 \sin^{-1} \left( \frac{6 - 2}{2(4)} \right) = 2.09 \text{ rad} = 120 \text{ deg} \tag{8.21}
\]

To find the tensile force \( F_1 \) that satisfies the motor torque constraint we use equation (8.16) to obtain

\[
F_1 = \frac{T}{\left( 1 - \frac{1}{e^{\phi_1}} \right)} = \frac{17.52}{1 - \frac{1}{e^{0.3(2.09)}}} = 37.5 \text{ lbs} \tag{8.22}
\]

We find the tension on the slack side of the belt as

\[
F_2 = \frac{F_1}{e^{\phi_1}} = \frac{37.5}{e^{0.3(2.09)}} = 20.0 \text{ lbs} \tag{8.23}
\]

The initial tension in the belt before the torque is applied is obtained as

\[
F_i = \frac{F_1 + F_2}{2} = \frac{37.5 + 20.0}{2} = 28.8 \text{ lbs} \tag{8.24}
\]
We check the belt torque using equation (8.15).

\[ T_b = (F_1 - F_2) r_1 = (37.5 - 20.0) \times 1 = 17.5 \text{ lb-in} \]  

(8.25)

We note that to satisfy the torque constraint, a 37.5-lb. tension will be necessary for \( F_1 \). But that level of tension exceeds the belt strength constraint of 35 lbs. Therefore, a center distance of 4 in. is an infeasible value. It causes a violation of a constraint and will need to be increased (to increase the angle of wrap, and consequently reduce the tension in the belt).

To reduce the effort in computing the expected system performance as a function of center distance, \( c \), we can develop a spreadsheet as shown in Table 8.4.

<table>
<thead>
<tr>
<th>Problem Definition Parameters</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>friction coefficient</td>
<td>( f )</td>
<td>none</td>
<td>0.3</td>
</tr>
<tr>
<td>belt strength</td>
<td>( F_{\text{max}} )</td>
<td>lbs</td>
<td>35</td>
</tr>
<tr>
<td>motor power</td>
<td>( W )</td>
<td>hp</td>
<td>0.5</td>
</tr>
<tr>
<td>motor speed</td>
<td>( n_1 )</td>
<td>rpm</td>
<td>1800</td>
</tr>
<tr>
<td>motor pulley diameter</td>
<td>( D_1 )</td>
<td>inches</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Symbol</th>
<th>Units</th>
<th>Lower</th>
<th>Value</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>driven pulley diameter</td>
<td>( d_2 )</td>
<td>inches</td>
<td>—</td>
<td>6</td>
<td>—</td>
</tr>
<tr>
<td>center distance</td>
<td>( c )</td>
<td>inches</td>
<td>4.0</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance Calculations</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
<th>Type</th>
<th>Value</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>motor torque</td>
<td>( T_m )</td>
<td>lb-in</td>
<td>17.51</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>grinding wheel speed</td>
<td>( n_2 )</td>
<td>rpm</td>
<td>600</td>
<td>=</td>
<td>600</td>
<td>Satisfied</td>
</tr>
<tr>
<td>angle of wrap</td>
<td>( \phi )</td>
<td>degrees</td>
<td>120.0</td>
<td>≤</td>
<td>35</td>
<td>Unsatisfied</td>
</tr>
<tr>
<td>belt tension-taut</td>
<td>( F_1 )</td>
<td>lbs</td>
<td>37.5</td>
<td>≤</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>belt tension-slab</td>
<td>( F_2 )</td>
<td>lbs</td>
<td>20.0</td>
<td>≥</td>
<td>17.51</td>
<td>Satisfied</td>
</tr>
<tr>
<td>initial belt tension</td>
<td>( F_i )</td>
<td>lbs</td>
<td>28.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>belt torque</td>
<td>( T_b )</td>
<td>lb-in</td>
<td>17.51</td>
<td>≥</td>
<td>17.51</td>
<td>Satisfied</td>
</tr>
</tbody>
</table>

**Redesigning: Finding Feasible Values** Since the belt tension is constrained to a maximum of 35 lbs., the initial value chosen for \( c \) is infeasible. We therefore increase the center distance and recalculate the belt tensions. Using the spreadsheet, we obtain the following values for belt tension, \( F_1 \) in Table 8.5.
TABLE 8.5 Belt Tension for Alternative Center Distances

<table>
<thead>
<tr>
<th>Center distance $c$ (in.)</th>
<th>Belt Tension $F_1$ (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>37.5</td>
</tr>
<tr>
<td>4.96 (Goalseek)</td>
<td>35.0</td>
</tr>
<tr>
<td>6</td>
<td>33.5</td>
</tr>
<tr>
<td>8</td>
<td>32.0</td>
</tr>
<tr>
<td>10</td>
<td>31.2</td>
</tr>
</tbody>
</table>

Using Excel’s Goalseek feature we find when $c \geq 4.96$ in., the belt-strength constraint is satisfied. However, to provide some extra capacity or compensate for some belt wear, or a slight decrease in friction coefficient, we usually select a larger value, perhaps 8, or even 10 in. Selecting $c = 8$ in., for example, we would obtain a 1.09 factor of safety $FS$, from:

$$FS = \frac{F_{\text{allowable}}}{F_{\text{design}}} = \frac{35}{32} = 1.09$$

**Factor of safety** is a term used to express ratio of loads or stresses as exemplified in equation (8.26). Factors of safety less than one mean that the design will not support the load without “failing.” Factors of safety greater than one indicate that the design is “safe.” Factors of safety are also discussed later in the chapter.

Finding feasible values for the design variables is what generating and analyzing is all about. **Infeasible designs** are those whose design variable values do not satisfy the constraints. We found that when $d_x = 2$ in. and $c = 8$ in. all the constraints are satisfied (speed, torque, and tension). But are these feasible values the best values?

**Trade-offs** As shown in Table 8.5, as we increase the center distance, we obtain lower values of belt tension, and consequently higher factors of safety. And that’s good. But, we are also increasing the size of the system. Unfortunately this design problem, like most design problems, exhibits a trade-off, wherein one attribute improves as the other degrades. Trade-offs are caused by the interdependency of variables, typically referred to as coupling. Both compactness and belt tension are coupled to the center distance. We trade-off higher belt tensions as we satisfy the customer’s desire for compactness. We could ask, “What is the best design?” Is more safety (lower belt tension) and a less compact system better than less safety and more compactness?

This question is difficult to answer. However, as it has been said, “beauty is in the eye of the beholder.” Why not consider the trade-offs from the customer’s perspective. That is, how does the customer feel about the importance of each of these attributes? Also, is there a way to assess overall satisfaction of the customer? This is the process of evaluating, which we consider in the next section.
### Evaluating

During evaluation, we choose the best design candidate from among those feasible designs that we generated and analyzed earlier. Often our company will want the least expensive design. Other times we may want to maximize an aspect of system performance to obtain the fastest, or the lightest, or the most fuel-efficient design. Like many design teams, we could, therefore, select one of these criteria as the measure of “best,” to determine our final recommendation.

Most design problems, however, have multiple attributes or criteria that usually involve trade-off decisions. In the belt-pulley system, for example, we need to make compromises between center distance and belt tension (i.e., compactness versus safety).

Therefore, we need a rational method that can help us make these necessary compromises among multiple criteria. As in concept design and configuration design, the weighted-rating method can be used and is recommended whenever possible.

**Step 1:** Establish a set of evaluation criteria.

**Step 2:** Rate the feasible designs for each criterion.

**Step 3:** Weight the ratings according to importance.

**Step 4:** Sum the weighted ratings to calculate an overall weighted rating.

---

**Step 1. Establish a set of evaluation criteria, importance weights, and satisfaction levels.** If we wish to maintain high standards of quality we need to incorporate the “voice of the customer.” Therefore, evaluation criteria are often developed from the list of solution evaluation parameters, or from engineering characteristics. Recall, that we use the house of quality, and/or other marketing research tools, to identify the customer’s: (1) functional requirements for the product, (2) key engineering characteristics (that measure how well the functions are performed), and (3) the importance of each functional requirement.

For our belt-pulley system, let’s assume that the product development team determined that as long as the motor horsepower was fully utilized, the two most important engineering characteristics were belt tension and center distance (compactness). We identified these earlier during the parametric design formulation step as solution evaluation parameters.

Let’s further assume that the team was also able to determine that the customer considers belt tension as “very important” and center distance as “important.” Assume that we interpret “very important” with a weight of 0.6 and “important” with a weight of 0.4. Note that we can certainly use other numerical values for importance levels. The very act of choosing numerical values often leads to lots of discussion among design team members. And while it may be time-consuming and frustrating, team discussion is very constructive in that it leads to a better understanding of the customer’s desires.

**Step 2. Rate the feasible designs for each criterion.** We can use the weighted rating method and the rating scale of 0–4 (unsatisfactory to very good), as we
used in Table 4.12. Note that the people doing the rating may or may not reflect the voice of the customer.

Rather than using a rating, we can use a customer satisfaction curve or function. We arbitrarily let minimum satisfaction be equal to the numerical value of 0 and maximum satisfaction be equal to the number 1. These are similar to the ratings of 0 and 4, discussed before. The main difference is that in satisfaction curves or functions, we link the value of each criterion, or SEP, to a value of satisfaction. This linking of satisfaction versus SEP values is facilitated using graphs and curve-fits. Let’s see how we can use these to evaluate our belt-pulley designs.

Let’s assume that the product development team also prepared estimates of customer satisfaction with respect to belt tension and center distance as graphed in Figures 8.7 and Figure 8.8.

### FIGURE 8.7 Customer satisfaction versus belt tension.

![Graph](image1.png)

### FIGURE 8.8 Customer satisfaction versus center distance.

![Graph](image2.png)

After the satisfaction curves are determined, we can rate each feasible design by reading values off the curves and entering them into the weighted-rating table.

To facilitate the weighted-rating calculations we can find analytical formulas that fit the curves (i.e., curve-fits) and then automate the computations.
using a spreadsheet. Basic formulas from geometry can be used as in our example. Special computer programs can be used. And Excel’s Trendline feature is quite capable in curve-fitting the following relationships: linear, polynomial, logarithmic, exponential, and power.

Since the curves are straight lines in this example, we can fit a simple, straight-line interpolation equation using the two-point formula

\[
(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)
\]  

(8.27)

We substitute satisfaction \( S \) for \( y \), and solution evaluation parameter for \( x \), noting that the maximum value of \( S \) is 1, and the minimum value of \( S \) is 0. For an SEP with a decreasing satisfaction curve we find the satisfaction as

\[
S_{x-} = \frac{(x_2 - x)}{(x_2 - x_1)}
\]  

(8.28)

For an SEP with an increasing satisfaction curve, we find the satisfaction as

\[
S_{x+} = \frac{(x - x_1)}{(x_2 - x_1)}
\]  

(8.29)

Note that equations (8.28) and (8.29) are only valid between \( x_2 \) and \( x_f \).

We set up the belt-pulley satisfactions in equations (8.30) and (8.31) for belt tension \( F_i \) as

\[
S_{F_i} = \frac{(35 - F_i)}{(35 - 30)}
\]  

(8.30)

and for compactness \( c \) as

\[
S_c = \frac{(20 - c)}{(20 - 5)}
\]  

(8.31)

**Step 3. Weight the rating according to importance.** In this step we multiply the satisfaction rating by the respective importance weight.

**Step 4. Sum the weighted ratings to calculate an overall weighted rating.** We can estimate the customer’s overall satisfaction \( Q \) as the sum of the importance-weighted satisfaction levels. Specifically, let \( S_i \) be the level of the customer’s satisfaction for \( i \)-th solution evaluation parameter and \( w_i \) be equal to the importance weight for that parameter. Then the overall customer satisfaction can be estimated by

\[
Q = \sum w_i S_i = w_1 S_1 + w_2 S_2 + ... 
\]  

(8.32)

For a center distance equal to 6 in. we can calculate the overall satisfaction as

\[
Q = \sum w_i S_i = w_{F_1} S_{F_1} + w_c S_c
\]  

(8.33)

\[
Q = 0.6 \frac{(35 - F_1)}{(35 - 30)} + 0.4 \frac{(20 - c)}{(20 - 5)}
\]  

(8.34)
Other values of overall satisfaction are calculated and summarized in Table 8.6. Note how values of the design variable, \( c \), are coupled to the values of belt tension and compactness. Also, note that equation (8.34) can only be used when both belt tension and center distance are within their respective interpolating points. Outside of these, a value of 0 or 1 will be multiplied by the respective importance weight.

The satisfaction values are also plotted in Figure 8.9. The satisfaction curve for belt tension illustrates how as \( c \) increases, the customer is more satisfied. This occurs because the belt tension decreases, resulting in higher factors of safety. And similarly, as \( c \) increases, the customer is less satisfied because the design is less compact.

The overall satisfaction curve, on the other hand, illustrates that the customer’s satisfaction is somewhat maximized when the center distance is about 10 in. Also, the figure indicates that the overall satisfaction is fairly insensitive from about \( c = 9 \text{ in.} \) to about \( c = 14 \text{ in.} \). That appears to indicate that the customer may be fairly indifferent to values of \( c \) between 9 and 14 in.

The overall satisfaction formula can be used as an aggregate objective function to optimize the belt-pulley design problem. An aggregate objective function is used to optimize problems when more than one criterion exists, in other words for multi-attribute optimization. Aggregate objective functions combine a number of separate objective functions into one scalar function.

\[
Q = 0.6 \left( \frac{35 - 33.5}{35 - 30} \right) + 0.4 \left( \frac{20 - 6}{20 - 5} \right) \\
Q = 0.6(0.3) + 0.4(0.93) = 0.18 + 0.37 = 0.55
\] (8.35) (8.36)

<table>
<thead>
<tr>
<th>Center distance ( c ) (in.)</th>
<th>Tension ( F_1 ) (lbs)</th>
<th>Tension ( S_{F_1} )</th>
<th>Compactness ( c ) (in.)</th>
<th>Compactness ( S_c )</th>
<th>Overall Satisfaction ( S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.96</td>
<td>35.00</td>
<td>0.00</td>
<td>4.96</td>
<td>1.00</td>
<td>0.40</td>
</tr>
<tr>
<td>6</td>
<td>33.50</td>
<td>0.30</td>
<td>6</td>
<td>0.93</td>
<td>0.37</td>
</tr>
<tr>
<td>8</td>
<td>32.00</td>
<td>0.60</td>
<td>8</td>
<td>0.80</td>
<td>0.32</td>
</tr>
<tr>
<td>10</td>
<td>31.25</td>
<td>0.76</td>
<td>10</td>
<td>0.67</td>
<td>0.27</td>
</tr>
<tr>
<td>14</td>
<td>30.40</td>
<td>0.92</td>
<td>14</td>
<td>0.40</td>
<td>0.16</td>
</tr>
<tr>
<td>18</td>
<td>30.00</td>
<td>1.00</td>
<td>18</td>
<td>0.13</td>
<td>0.05</td>
</tr>
<tr>
<td>20</td>
<td>29.90</td>
<td>1.00</td>
<td>20</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Using Excel’s Solver feature and the overall satisfaction formula as an aggregate objective function, an optimal value of \( c = 11.38 \) in. resulted in an overall satisfaction of 0.723, whereas \( c = 10 \) in. results in an overall satisfaction of 0.717. The set of the design variable values that satisfies all the constraints (speed, torque, and tension) and maximizes the customer’s satisfaction is therefore, \( d_2 = 2 \) in. and \( c = 11.38 \) in.

8.4 SYSTEMATIC PARAMETRIC DESIGN: STORAGE TANK EXAMPLE

Let’s consider a classical problem that continues the use of spreadsheets but also involves using the necessary and sufficient conditions for finding the optimum of a function of one variable.

8.4.1 Design Problem Formulation

Our company’s Canadian office has asked our engineering team to design an above-ground fuel storage tank. The tank should hold a minimum of 10,000 liters. However, they would be very happy if they could afford a tank holding
15,000 liter or more. Their budget is limited to a maximum of $10,000, but would be very satisfied if the tank would cost less than $5,000.

The design team has also determined that:

- local codes require sturdy steel walls at least 10 mm thick,
- to facilitate routine maintenance the tank should be cylindrical,
- two half-cylinder shells would be rolled then welded together,
- two circular plates would be welded to the top and bottom,
- building codes restrict height to a maximum of 15 meters,
- wall thickness is negligible compared to diameter for calculating purposes,
- steel weighs 7,830 kg/m³,
- steel sheet costs about $3/kg,
- footprint should be no larger than 10 meters square,
- welding, rolling and other incidental costs are insignificant, and
- cost and volume have a importance weights of 65%, 35% respectively.

**Solution Evaluation Parameters (SEPs)** The principal function of the tank is to safely store fuel. The walls must be sturdy to resist bucking, corrosion and local weather conditions. Our affiliate’s satisfaction will depend on getting a tank with as much volume $V$ as possible. It will also be more satisfied if cost $C$ is controlled.

For evaluation purposes we will assume that they would be 100% satisfied if the volume were 15,000 liters or more. They would be not be satisfied with anything less than 10,000 liters, nor with a cost greater than $10,000. We can assume that they would be satisfied with a cost of $5,000 or less. The solution evaluation parameters are summarized in Table 8.7. The satisfaction curves are presented in Figure 8.10 and Figure 8.11. Note that the satisfaction curve for cost is a “less-is-better” shape and that the satisfaction for volume is a “more-is-better” shape. Having both “less-is-better” and “more-is-better” satisfaction curves usually means that we will have a trade-off to make. For example we may trade a higher cost tank (resulting in less cost satisfaction), to gain more storage volume (resulting in more storage satisfaction).

We can estimate the customer’s overall satisfaction $Q$ as the sum of the importance-weighted satisfaction levels for cost and volume as

$$Q = \sum w_i S_i = w_C S_C + w_V S_V$$

(8.37)

**TABLE 8.7 Solution Evaluation Parameters for Storage Tank**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>cost</td>
<td>$C$</td>
<td>$</td>
<td>$5,000</td>
</tr>
<tr>
<td>2</td>
<td>volume</td>
<td>$V$</td>
<td>liter</td>
<td>10,000</td>
</tr>
</tbody>
</table>
FIGURE 8.10 Satisfaction as a function of storage tank cost. Note that satisfaction is largest for a tank costing less than $5,000.

FIGURE 8.11 Satisfaction as a function of storage tank volume. Note that satisfaction is largest for tanks bigger than 15,000 liters.

\[ Q = 0.65 \frac{(10,000 - C)}{(10,000 - 5,000)} + 0.35 \frac{(V - 10,000)}{(15,000 - 10,000)} \]  

(8.38)

Equation (8.38) should be used with caution. The linear interpolation formula can only be used between the end points. This can be handled with an appropriate nested “if” statement for each term, for example:

=IF(cost <=5000,.65,IF(cost>10000,0,.65*(10000-cost)/(10000-5000))).

**Design Variables (DVs)** The two rolled half-shells are shown in Figure 8.12. The top and bottom circular disks are not shown. The radius \( r \), thickness, \( t \) and height \( h \) are variables that determine the cost of the tank. Local codes restrict the wall thickness to 10 mm. Therefore the controllable variables are radius and height. They are therefore defined as the design variables as shown in Table 8.12.
Problem Definition Parameters (PDPs)  The givens for this design problem include the unit cost, density of steel and wall thickness (local standard). We identify these as the problem definition parameters in Table 8.9.

Plan for Solving  We will use a number of analytical relations from math and physics to estimate values of cost and volume as a function of radius and height. Then we will check that the constraints are not violated. Specifically, we will make sure that the thickness is exactly 10 mm, the minimum volume of 10,000 liters, the footprint is less than 10 (m$^2$) and the $15,000 maximum cost are not violated.
8.4.2 Generating, Analyzing and Evaluating

We can model the tank with a system of equations and thereby analyze the performance of each set of design variable values, which we will call design candidates.

**Modeling** For example, we know that the material volume $V_M$ of the tank walls, top and bottom is the product of area $A$ and thickness $t$. Further, the weight $W$ is the product of the volume and the weight density $\gamma$ as

$$W = \gamma V_M = \gamma t A$$  \hspace{1cm} (8.39)

The area of the tank walls, top and bottom can be found from

$$A = A_{walls} + A_{top} + A_{bottom}$$  \hspace{1cm} (8.40)

$$A = h(2\pi r) + \pi r^2 + \pi r^2$$  \hspace{1cm} (8.41)

Which simplifies to

$$A = 2\pi (hr + r^2)$$  \hspace{1cm} (8.42)

The weight and cost are consequently modeled as

$$W = \gamma V_M = 2\pi \gamma t (hr + r^2)$$  \hspace{1cm} (8.43)

$$C = c_M W$$  \hspace{1cm} (8.44)

$$C = c_M 2\pi \gamma t (hr + r^2)$$  \hspace{1cm} (8.45)

The storage volume, $V$ can be calculated from the formula for the volume of a right circular cylinder as

$$V = h\pi r^2$$  \hspace{1cm} (8.46)

From equation (8.45) and (8.46) we see that cost and volume are directly related to the radius and the height. We note that this design problem involves two equations and two unknowns. For a tank of constant volume, we can eliminate height, $h$ by substituting equation (8.46) into (8.45) obtaining

$$C = c_M 2\pi \gamma t ((V / \pi r^2) r + r^2)$$  \hspace{1cm} (8.47)

Which simplifies to

$$C = c_M 2\pi \gamma t \left(\frac{V}{\pi r} + r^2\right)$$  \hspace{1cm} (8.48)

Since all the parameters in equation (8.48) are constants except for radius, it is a function of one variable. Using the necessary condition from calculus for determining a stationary point we can take the first derivative of the cost function with respect to the radius and set it to zero to obtain

$$\frac{dC}{dr} = \frac{d}{dr} \left(c_M 2\pi \gamma t \left(\frac{V}{\pi r} + r^2\right)\right) = 0$$  \hspace{1cm} (8.49)
\[
\frac{dC}{dr} = c_M 2\pi \gamma t \left( -\frac{V}{\pi r^2} + 2r \right) = 0
\]

(8.50)

Finding the optimal radius as

\[
r^* = \left( \frac{V}{2\pi} \right)^{1/3}
\]

(8.51)

We next use the sufficient condition for an optimum to test the second derivative as

\[
\frac{d^2C}{dr^2} = \left( \frac{2V}{\pi r^3} + 2 \right) > 0
\]

(8.52)

Since the second derivative of the cost function is greater than zero for positive, non-zero values of \(r\), we readily conclude that the stationary point \(r^*\) is a minimum. This behavior is illustrated in Figure 8.13.

**Analyzing and Evaluating** Recognizing that a minimum weight tank can be designed using the optimal radius and height, we can develop a spreadsheet to compute the weight, cost and weighted satisfactions as a function of volume.

As shown in Table 8.10, we see that as the volume of the tank increases, the weight and cost increase. The weighted satisfaction for cost decreases as the tank becomes more expensive. If we were to merely minimize cost, we would select a 10,000 liter design with radius 1.17 (m) and height 2.34 (m). This decision would result in an overall satisfaction of 0.52. The weighted satisfaction for volume increases with larger tank sizes reaching 0.35 for tank sizes of 15,000 (liter) or more. Note how these trade-off. Which tank size is the “best?”

![Cost versus Radius](image)

**FIGURE 8.13** The cost for a 10,000 liter tank reaches a minimum for a radius of 1.17 (m). The optimal radius \(r^*\) can be calculated using equation (8.51).
If we were to make our final decision solely based on a single attribute, we would select a 10,000 (liter) tank size. To maximize total satisfaction we should use overall satisfaction \( Q \), which is a sum of the two weighted satisfaction values. Overall satisfaction reaches a maximum of 0.62 at a tank size of 15,000 (liter). This reflects the increase in volume for a marginal increase in cost. A tank larger than 15,000 (liter), however, does not improve satisfaction because the customer is indifferent (no change in satisfaction) for tanks larger than 15,000 (liter). Therefore we select a 15,000 (liter) tank with radius 1.34 (m), height 2.67 (m) and a cost of $7,090. These trends are illustrated in Figure 8.14.

Lastly, we must also check that constraints are not violated. The thickness is equal to 10 (mm). The cost is less than $15,000. The tank volume is greater than 10,000 (liter). And the footprint is \( \pi r^2 = \pi (1.34)^2 = 5.6 \) (m²).

Our selected design maximizes satisfaction and does not violate constraints.
FIGURE 8.14 Weighted satisfaction for cost decrease as tank size increases. Similarly the weighted satisfaction for volume increases with tank size. The overall satisfaction reaches a maximum for a tank size of 15,000 liters.

8.5 DESIGN FOR ROBUSTNESS

A robust product is one whose performance is insensitive to variations. Variations come from a variety of sources including: manufacturing, wear, and the operating environment. When a product is manufactured, minor variations in material composition can occur. Sometimes an alternative, less costly, material with similar mechanical properties is purchased. In other cases, we might heat-treat or surface-coat the material in manufacturing processes that have resulting variations. And, as important, most of the manufacturing processes, such as machining, casting, and sheet metal stamping, will produce variations in final dimensions.

As the product is used, its moving parts undergo abrasive and adhesive wear. Also, some materials have properties that degrade over time. Steels become brittle with exposure to hydrogen gases and some polymers become opaque, etc. Then, we might also consider situations when the customer abuses the product, by subjecting it to higher-than-expected loads.

The operating environment can also cause variations in performance. Excessive humidity, dust, dirt, and extremely hot or cold environments can alter product performance. Steel, for example, can be brittle at sub-zero. Corrosive gases or vapors, such as in chemical plants or food-processing plants, can also cause performance degradation.

Design for robustness is a term used to describe a number of methods aimed at reducing the sensitivity of product performance to variations (Fowlkes
and Creveling, 1995). During parametric design, these variations in material properties, dimensions, and operating conditions can be simulated. Alternative designs can be generated and analyzed for their sensitivity to these variations.

Two methods used quite frequently are probabilistic optimal design and the Taguchi method. Probabilistic optimal design uses methods such as Monte Carlo simulation combined with nonlinear optimization algorithms to design parts with high levels of reliability (Eggert and Mayne, 1993; Haugen, 1980; Rao, 1984; Siddall, 1983). The Taguchi method exploits the use of statistics from design of experiments to reduce the sensitivity of products or processes to “noise” (i.e., variations) (Ross, 1996).

8.6 COMPUTER-AIDED ENGINEERING

Computer-aided engineering (CAE) refers to computer software and hardware systems used in the analysis of engineering designs to validate functional performance. Computer-aided design (CAD) is often used interchangeably with CAE.

CAE systems can be separated into four main categories: dynamics analysis, finite element analysis (FEA), general purpose, and other.

Dynamics analyses involves the kinematics and kinetics of bodies, that is, the motion and the forces and torques that cause motion. We find that solid-modeling packages, for example, will mate parts into an assembly, and that we can manipulate various parts to animate their relative motion. However, in a dynamics analysis package such as Working Model, ADAMS, or DADS, we apply specified loads to the parts and the package will calculate their resultant motion using fundamental equations of physics and a variety of numerical methods. These packages simulate behavior based on physics, whereas CAD solid-modeling packages animate motion, independent of physics. Since kinematics and kinetics-based packages are based on the physics of the problem, they can analyze a number of behaviors including:

- positions, velocities, accelerations,
- contacts and collisions,
- joint forces, shaking forces, and
- relative motions.

Finite element analysis (FEA) is a method that essentially divides a part into smaller discrete elements to analyze the functional performance of a part. Used in many engineering fields, the process begins with the creation of a geometric model using CAD software. Solid-modeling packages can export files to FEA packages and some can communicate directly with FEA packages. The part is usually subdivided into a mesh of smaller finite elements connected at nodes. Loads are then virtually applied to the part. Using fundamental equations from physics and engineering, an FEA package can analyze a number of behaviors including:
- stresses and strains throughout the part,
- factors of safety,
- buckling of parts in compression,
- temperature gradients and resulting heat transfer,
- fluid pressures and flows,
- natural frequencies of vibration, and
- displacements of nodes.

General-purpose software refers to computer applications covering word processing, spreadsheets, mathematics, oral presentations, and project management. We spend a significant amount of time communicating our work to others within and without our department and/or company. Having applications such as MSWord®, Excel®, MathCAD®, PowerPoint®, and MSProject® can help us complete these daily tasks more efficiently.

Other computer-aided engineering applications include:

- Quality function deployment—applications that facilitate the development of houses of quality such as QFD Capture.
- Material selection—specialized software that aids in screening materials such as GRANTA design, which implements Ashby’s methodology.
- DFMA—applications that facilitate design for assembly and/or design for manufacture analyses such as the Boothroyd-Dewhurst package.
- Systems simulation—such as MATLAB Simulink, which analyzes the transient and steady-state behavior of multi-component electrical and mechanical systems.
- Variance analysis—applications that compute tolerance stacks in assemblies.

8.7 SUMMARY

- The parametric design phase includes decision-making processes to determine design variable values that satisfy the constraints and maximize the customer’s satisfaction.
- The five steps in parametric design are: (1) formulate, (2) generate, (3) analyze, (4) evaluate, and (5) refine/optimize.
- During parametric design we ensure that all the candidates are feasible.
- Many design problems typically exhibit trade-off behavior, requiring compromises.
- Customer satisfaction curves and or functions can be used to determine the best candidate from among the feasible design candidates.
- Design for robustness methods decrease the sensitivity of candidate designs to variations due to materials, manufacturing, or operating conditions.
- Computer-aided engineering applications are useful in analyzing the performance of candidate designs.
REFERENCES


KEY TERMS

- Aggregate objective function
- Computer-aided engineering
- Constraint
- Coupling
- Design variable
- Factor of safety
- Feasible design
- Finite element Analysis (FEA)
- Infeasible design
- Inverse analysis
- Multiattribute optimization
- Objective function
- Optimize
- Parametric design
- Problem definition parameter
- Robust product
- Solution evaluation parameter
- Trade-off

EXERCISES

Self-Test. Write the letter of the choice that best answers the question.

1. _____ We complete product specifications, drawings, performance tests, and bills of materials in:  a. detail design  b. configuration design  c. parametric design  d. concept design
2. _____ We make decisions about the physical principles, geometry, and material in the:  
   a. detail design phase  
   b. configuration design phase  
   c. parametric design phase  
   d. concept design phase

3. _____ We decide the type of geometric features on a part, their arrangement and their relative dimensions in:  
   a. detail design  
   b. parametric design  
   c. configuration design  
   d. concept design

4. _____ In which parametric design step do we assess feasible designs to determine the best design:
   a. analyze  
   b. refine  
   c. evaluate  
   d. generate

5. _____ In which of the five steps of parametric design do we gather appropriate data, clarify important details, consider different analytical and experimental analysis, and then plan and decide how to complete the project:
   a. generate  
   b. analyze  
   c. evaluate  
   d. formulate

6. _____ In which of the five steps of parametric design do we select different values for the design variables to synthesize different candidate designs:
   a. formulate  
   b. generate  
   c. evaluate  
   d. refine

7. _____ Values of the design variables that do not satisfy the constraints are said to be:
   a. feasible  
   b. nonoptimal  
   c. inaccurate  
   d. infeasible

8. _____ Design variables are parameters that:
   a. describe specific conditions of use  
   b. are under the control of the design engineer  
   c. measure product performance  
   d. measure customer satisfaction

9. _____ Methods aimed at reducing the sensitivity of product performance to variations are called:
   a. weighted rating  
   b. design evaluation  
   c. design for robustness  
   d. optimal performance

10. _____ Methods to automate the generation of design variable values are:
    a. design for robustness  
    b. safe design  
    c. optimal design  
    d. synthesis

11. _____ Software that analyzes the motion and the forces and moments that cause motion:
    a. CAD  
    b. dynamics  
    c. FEA  
    d. QFD

12. _____ Trade-offs result in SEP compromises because of:
    a. coupling  
    b. variation  
    c. errors  
    d. constraints

13. _____ Finding the values of design variables by algebraically juggling an equation is called:
    a. QFD  
    b. inverse analysis  
    c. variation analysis  
    d. optimization

14. _____ A factor of safety for loads is the ratio of allowable load divided by:
    a. desirable force  
    b. design load  
    c. maximum load  
    d. minimum load

15. _____ Constraint can be all of the following except:
    a. limits on design variables  
    b. inequality relations  
    c. equality relations  
    d. PDPs

16. As discussed in the bolt design example in the text, the equation for area can be “juggled” or rearranged to explicitly solve for diameter $d$. Many engineering problems involve implicit relations such as the Secant buckling formula. Explain how you might solve for area $A$, if given other variable values. (Do not solve).

\[
P_{cr} = \frac{ASy}{1 + (ec/\rho^2)\sec[(Le/\rho)\sqrt{P_{cr}/4AE}]}\]
17. A customer would like to support a 2,000-pound art sculpture on a column 10 feet high. You have been hired to complete the parametric design. The customer would be unhappy if the column buckled or cost too much. Your column, he further suggests, should be able to support a design load of 6,000 pounds. Begin the parametric design by formulating the problem and complete the following tables. List pertinent constraints. Do we know if cost or safety is more important to the customer? Why? Should cost or safety be more important to the design engineer? Other problem information includes:

- $E_{\text{aluminum}} = 10 \text{ Mpsi}, E_{\text{steel}} = 30 \text{ Mpsi}$.
- The force causing the first sign of buckling is the critical load $P_{cr}$ according to Euler’s buckling formula below, $E = \text{modulus of elasticity}, I = \text{moment of area (inertia)}, L = \text{column length}$.

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

- Column available in different cross sections: circular, rectangular, box, and structural “H.”
- Column cost is proportional to moment of inertia $I$ and can be approximated by the following relation: $C = 35.6 I [\$]$.

### Solution Evaluation Parameter(s)

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### Design Variable(s)

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### Problem Definition Parameter(s)

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18. Parametric design mini project.
A cylindrical chemical storage tank is closed at both ends. It has diameter $D$ and height $H$. The tank must store at least 150 m$^3$ of liquid. It is made of rolled and welded sheet steel whose material and fabrication costs are $550/m^2$ of surface area. The height is restricted to 10 m, and the diameter must be less than 5 m. The customer will be satisfied if the tank cost is small and the volume is large. He feels that cost is very important (weight = 0.7) and that volume is important (weight = 0.3). Your design team has estimated that his satisfaction for cost is 1.0 when cost is equal to, or less than, $50,000, and it is 0.0 when cost is more than $100,000. His satisfaction for volume is 0.0 for volumes equal to or less than 130 m$^3$, and 1.0 for volumes greater than 180 m$^3$. Straight-line equations can be used to estimate his satisfactions for values of cost or volume in between.

a. Prepare an Excel spreadsheet to analyze and evaluate your candidate designs.
Label design variables, solution evaluation parameters(s), and problem definition parameter(s), as well as any other items. Label the variable names, symbol, units, and upper and lower limits, if any.

b. Write all the formulas you developed on a separate sheet of engineering paper, with your name and date, to be attached to your spreadsheet. Use a straight-line equation for each satisfaction function (i.e., for cost and volume). A convenient way of automating the satisfaction function in Excel is to use a nested “IF” statement, such as: $=\text{IF}($cost $< 50000, 1, \text{IF}($cost $> 100000, 0, (100000-\text{cost})/(100000-50000))$)

c. Use your spreadsheet to analyze and evaluate points 1–6 given below. Write in your spreadsheet results on the attached summary table.

d. Examine your results. Is there any pattern? Generate three more candidate designs, by guessing and/or logical reasoning to guide your redesign. Print out a copy of the spreadsheet for the last design point tried.

e. Circle the design alternative/point on the summary table that has the “best” weighted satisfaction. (Note: a co-worker says that he can find values for $D$ and $H$ to optimize satisfaction to about 0.334.)

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Answers to even numbered Self-Test questions: 2 d, 4 c, 6 b, 8b, 10c, 12a, 14b